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## David Taylor Research Center

Bethesda, MD 20084-5000

**AD-A221 071**

**DTRC-PAS-90/22** March 1990

Propulsion and Auxiliary Systems Department  
Research & Development Report

### On the Relationship Between Energy Density and Net Power (Intensity) in Coupled One-Dimensional Dynamic Systems

by  
G. Maidanik  
J. Dickey

On the Relationship Between Energy Density and Net Power  
(Intensity) in Coupled One-Dimensional Dynamic Systems

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## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			Approved for public release; distribution is unlimited.	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)  DTRC-PAS-90/22			5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION  David Taylor Research Center		6b. OFFICE SYMBOL (If applicable)  Code 2704	7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code)  Annapolis, MD 21402			7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION  David Taylor Research Center		8b. OFFICE SYMBOL (If applicable)  Code 1903	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)  Bethesda, MD 20084-5000			10. SOURCE OF FUNDING NUMBERS	
			PROGRAM ELEMENT NO.  62323N	PROJECT NO.  
			TASK NO.  	WORK UNIT ACCESSION NO.  
11. TITLE (Include Security Classification)  On the Relationship Between Energy Density on a Net Power (Intensity) in Coupled One-Dimensional Dynamic Systems				
12. PERSONAL AUTHOR(S)  G. Maidanik and J. Dickey				
13a. TYPE OF REPORT  Final		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (YEAR, MONTH, DAY)  1990 May
15. PAGE COUNT  8				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP		
			Acoustic intensity; Acoustic power flow; Structural intensity; Structural vibration.	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) A formalism for the power flow vector of a complex composed of a multitude of one-dimensional dynamic systems has been developed using wave propagation concepts. Each element in the power flow vector specifies the power flow in a given dynamic system. In this formalism, the complex is defined in terms of two diagonal propagator matrices, two terminal position vectors, and two junction matrices. An element in a propagator matrix describes the manner in which power, in a specific dynamic system, propagates toward a junction. An element in a terminal vector defines the terminal position, in a specific dynamic system, at a junction. A junction is the boundary that defines the coupling among the dynamic systems at a common terminal vector. An element in a junction matrix defines either the coupling (transmission action) between two distinct dynamic systems or the self-coupling (reflection action) at the designated junction. It is shown that the formalism accounts for the energetics of coupled one-dimensional dynamic systems. Neglecting cross terms between linear propagation toward one and the other junction, one may show that the stored energy density vector is the sum of the stored energy density vector associated with propagation of power toward one junction and with that propagating toward the other junction. Under the same conditions, one may further show that the net power (intensity) vector is the difference between the power vector associated with power propagation toward one junction and with that propagating toward the other. Since the energy stored and the power flow are simply related by a speed of propagation, it is argued that the stored energy density vector and the net power (intensity) vector are supplemental quantities. <i>Key words.</i>				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION  Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL  J. Dickey			22b. TELEPHONE (Include Area Code)  (301) 267-2759	22c. OFFICE SYMBOL  Code 2704

DD FORM 1473, JUN 86

Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

0102-LF-014-6602

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## ABSTRACT

A formalism for the power flow vector of a complex composed of a multitude of one-dimensional dynamic systems has been developed using wave propagation concepts. Each element in the power flow vector specifies the power flow in a given dynamic system. In this formalism, the complex is defined in terms of two diagonal propagator matrices, two terminal position vectors, and two junction matrices. An element in a propagator matrix describes the manner in which power, in a specific dynamic system, propagates toward a junction. An element in a terminal vector defines the terminal position, in a specific dynamic system, at a junction. A junction is the boundary that defines the couplings among the dynamic systems at a common terminal vector. An element in a junction matrix defines either the coupling (transmission action) between two distinct dynamic systems or the self-coupling (reflection action) at the designated junction. It is shown that the formalism accounts for the energetics of coupled one-dimensional dynamic systems. Neglecting cross terms between linear propagation toward one and the other junctions, one may show that the stored energy density vector is the sum of the stored energy density vector associated with propagation of power toward one junction and with that propagating toward the other junction. Under the same conditions, one may further show that the net power (intensity) vector is the difference between the power vector associated with power propagation toward one junction and with that propagating toward the other. Since the energy stored and the power flow are simply related by a speed of propagation, it is argued that the stored energy density vector and the net power (intensity) vector are supplemental quantities.

## ADMINISTRATIVE INFORMATION

This work was sponsored in part by the Submarine Technology Block (ND3A) 6.2 Submarine Silencing Task RB 23 C 33. The cognizant program manager is Mr. G. Smith, DTRC Code 1903.

## INTRODUCTION

The equations for the energetics of a complex are derived in a manner that parallels the derivation of the equation for the linear response of a corresponding complex. The equation for the linear response of a complex composed of coupled one-dimensional dynamic systems was previously developed and reported in References 1 through 3. Earlier attempts to utilize wave concepts to derive the equation for the energetic responses of such complexes were also reported;

for example, in References 4 through 6. In this paper further attempts in this utilization are reported and discussed. The derivation of the equation for the energetics of a complex is still restricted to coupled one-dimensional dynamic systems. The one dimensionality introduces considerable simplification. This simplification is used here to explain a number of relationships that would otherwise be lost in the cumbersomeness of the mathematical manipulations and the notations that would ensue were one to increase the spatial dimensionality.

### ANALYTICAL DEFINITION OF THE COMPLEX

A complex consisting of a number of one-dimensional dynamic systems is depicted in Figure 1. A dynamic system, the (j)th, is defined in terms of two propagators  $\lambda_{\infty j}^{\alpha}(x_j | x'_j)$ , two terminal positions  $x_{\alpha j}$ , and two junction vectors  $\Gamma_{\alpha j} = \{\Gamma_{\alpha ij}\}$ ;  $\alpha = r$  and  $q$ ; see Figure 2. The propagators  $\lambda_{\infty j}^{\alpha}(x_j | x'_j)$  define the forward propagation of power in the direction of junction  $\alpha$  from position  $x'_j$  to position  $x_j$  in the (j)th dynamic system were it extrapolated beyond its boundaries in a manner that would not back-scatter from the extrapolated regions. [The backward propagation in this process is considered absent.] The terminal positions  $x_{\alpha j}$  define the extent of the dynamic system; see Figure 2, each terminal position is at a junction. A junction is a "boundary" at which the dynamic systems interact with each other (transmission) and with itself (reflection):  $\Gamma_{\alpha ij}$ , with  $i \neq j$ , is the transmission coefficient of power (transmission action) at the ( $\alpha$ )th junction from the (j)th dynamic system into the (i)th dynamic system and  $\Gamma_{\alpha jj}$  is the reflection coefficient of power (reflection action) at the ( $\alpha$ )th junction in the (j)th dynamic system. One may then define the entire complex in terms of the power propagator matrices<sup>1</sup>

$$\underline{\lambda}_{\infty}^{\alpha}(x | x') = (\lambda_{\infty j}^{\alpha}(x_j | x'_j) \delta_{ji}) \quad , \quad \alpha = r \text{ and } q \quad , \quad (1)$$

---

<sup>1</sup>The dependence of quantities on either the temporal or the frequency variable is not stated explicitly throughout this paper. The explicit dependence may be readily included, however, at some increase in definitions and cumbersomeness. Since the inclusion of one form or the other does not change the general conclusions, it is decided to leave the dependence neuter and inexplicit.

the terminal position vectors

$$\underline{x}_\alpha = \{x_{\alpha j}\} \quad , \quad \alpha = r \text{ and } q \quad , \quad (2)$$

and the power junction matrices

$$\underline{\Gamma}_\alpha = (\Gamma_{\alpha ji}) \quad , \quad \alpha = r \text{ and } q \quad . \quad (3)$$

### COMPATIBLE DEFINITION OF THE EXTERNAL DRIVES

The external drive need be compatibly defined in terms of powers. The external drive on the (j)th dynamic system is stated in terms of two input powers: the external drive initiates the power flow  $\hat{\alpha}_j \pi_{e\alpha j}(x'_j)$  at position  $(x'_j)$  in the direction of junction  $\alpha$ ;  $\alpha = r$  and  $q$ , where  $\hat{\alpha}_j$  is a unit vector in this direction;  $[\hat{\alpha}_j \hat{\beta}_j] = \delta_{\alpha\beta}$ . The input power vectors may then be defined

$$\begin{aligned} \hat{\pi}_e(x') &= \sum_{r, q} \underline{\alpha} \pi_{e\alpha}(x') \quad ; \quad \pi_{e\alpha}(x') = \{\pi_{e\alpha j}(x'_j)\} \quad ; \\ \underline{\alpha} &= (\hat{\alpha}_j \delta_{ji}) \quad ; \quad [\underline{\alpha} \underline{\beta}] = (\delta_{\alpha\beta} \delta_{ji}) \quad . \end{aligned} \quad (4)$$

### DERIVATION OF THE IMPULSE RESPONSE MATRIX OF POWER AND THE POWER FLOW VECTORS

Assisted by Figure 3 and equations (1) through (4), one may state that

$$\hat{\lambda}^\alpha(x|x') = \underbrace{\hat{\lambda}_\infty^\alpha(x|x') \underline{\alpha}}_{\text{Direct}} + \underbrace{\hat{\lambda}_\infty^\alpha(x|x_\beta) \underline{\Gamma}_\beta \hat{\lambda}^\beta(x_\beta|x')}_{\text{Reverberant}} \quad , \quad (5)$$

$$\pi^\alpha(x) = \int [\hat{\lambda}^\alpha(x|x') d\underline{x}' \hat{\pi}_e(x')] \quad ; \quad d\underline{x}' = (dx'_j \delta_{ji}) \quad , \quad (6)$$

or even

$$\pi^\alpha_\beta(x) = \int [\hat{\lambda}^\alpha(x|x') d\underline{x}' \underline{\beta} \pi_{e\beta}(x')] \quad , \quad (7)$$

where  $\hat{\underline{\underline{\lambda}}}^\alpha(\underline{x} | \underline{x}')$  is the impulse response matrix of power, and  $\underline{\pi}^\alpha(\underline{x})$  is the power flow vector at  $\underline{x} = \{x_j\}$  that is directed toward junction  $\alpha$ . The quantity  $\underline{\pi}_\beta^\alpha(\underline{x})$  is the portion of that power flow vector that is initiated by an injection of external power that is directed toward junction  $\beta$ . [Note that  $\underline{\pi}^\alpha(\underline{x}) = \underline{\pi}_\infty^\alpha(\underline{x}) + \underline{\pi}_\alpha^\alpha(\underline{x}) + \underline{\pi}_\beta^\alpha(\underline{x})$ , where  $\underline{\pi}_\infty^\alpha(\underline{x})$  is the direct term and  $\underline{\pi}_\alpha^\alpha(\underline{x}) + \underline{\pi}_\beta^\alpha(\underline{x})$  are the reverberant terms in  $\underline{\pi}^\alpha(\underline{x})$ ; see equation (5). The direct term  $\underline{\pi}_\infty^\alpha(\underline{x})$  is the power flow vector that is associated with propagation toward junction  $\alpha$  prior to any interaction with the junctions.] From equation (5), the impulse response matrix of power appears to be impure;  $\hat{\underline{\underline{\lambda}}}^\alpha(\underline{x} | \underline{x}')$  is not explicitly expressed in terms of the three pairs of quantities and parameters that describe the complex; namely, equations (1) through (3). Cumbersome but straightforward manipulations of equation (5), however, makes the purity of  $\hat{\underline{\underline{\lambda}}}^\alpha(\underline{x} | \underline{x}')$  explicit. Such manipulations yield

$$\hat{\underline{\underline{\lambda}}}^\alpha(\underline{x} | \underline{x}') = [\hat{\underline{\underline{\lambda}}}_\infty^\alpha(\underline{x} | \underline{x}') + \hat{\underline{\underline{\lambda}}}_\alpha^\alpha(\underline{x} | \underline{x}')] \underline{\Gamma}_\alpha + \hat{\underline{\underline{\lambda}}}_\beta^\alpha(\underline{x} | \underline{x}') \underline{\underline{D}}_\beta, \quad (8)$$

$$\hat{\underline{\underline{\lambda}}}_\alpha^\alpha(\underline{x} | \underline{x}') = \lambda_\infty^\alpha(\underline{x} | \underline{x}_\beta) \underline{\Gamma}_\beta \underline{\underline{D}}_\beta \underline{\Gamma}_\alpha^\beta \hat{\underline{\underline{\lambda}}}_\infty^\alpha(\underline{x}_\alpha | \underline{x}') , \quad (9a)$$

$$\hat{\underline{\underline{\lambda}}}_\beta^\alpha(\underline{x} | \underline{x}') = \lambda_\infty^\alpha(\underline{x} | \underline{x}_\beta) \underline{\Gamma}_\beta \underline{\underline{D}}_\beta \hat{\underline{\underline{\lambda}}}_\infty^\beta(\underline{x}_\beta | \underline{x}') , \quad (9b)$$

where<sup>2</sup>

$$\underline{\underline{B}}_\beta = [\underline{I} - \underline{\Gamma}_\alpha^\beta \underline{\Gamma}_\beta^\alpha] ; \quad \underline{\Gamma}_\alpha^\beta = \hat{\underline{\underline{\lambda}}}_\infty^\beta(\underline{x}_\beta | \underline{x}_\alpha) \underline{\Gamma}_\alpha ; \quad \underline{\underline{D}}_\beta = (\underline{\underline{B}}_\beta)^{-1}. \quad (10)$$

From equations (8) through (10), it is clear that the impulse response matrix  $\hat{\underline{\underline{\lambda}}}^\alpha(\underline{x} | \underline{x}')$  of power, explicitly stated in equations (8) and (9), is pure. Therefore, the power flow vectors defined in equations (6) and (7) are properly defined and stated.

<sup>2</sup>The quantity  $\underline{\underline{B}}_\beta$  may be a temporal (or even a frequency) matrix operator. Its inversion into  $\underline{\underline{D}}_\beta$ , and the subsequent insertion of  $\underline{\underline{D}}_\beta$  into the reverberant terms in the equations for the impulse response matrix, cannot be made cavalierly when these equations are to be used.



## ENERGY DENSITY, INTENSITY, AND NET POWER VECTORS

The power flow vector  $\pi^\alpha(\underline{x})$  may be directly related to the energy density vector  $\underline{\varepsilon}^\alpha(\underline{x})$  and the intensity vector  $\underline{l}^\alpha(\underline{x})$  by the following equations:

$$\underline{\varepsilon}^\alpha(\underline{x}) = (\underline{C}^\alpha)^{-1} \pi^\alpha(\underline{x}) ; \quad \underline{C}^\alpha = (C_j^\alpha \delta_{ji}) , \quad (11)$$

$$\underline{l}^\alpha(\underline{x}) = (\underline{A}^\alpha)^{-1} \pi^\alpha(\underline{x}) ; \quad \underline{A}^\alpha = (A_j^\alpha \delta_{ji}) , \quad (12)$$

respectively, where  $C_j^\alpha$  is the speed of propagation toward junction  $\alpha$  in the  $(j)$ th dynamic system, and  $A_j^\alpha$  is the cross-section facing junction  $\alpha$  in the  $(j)$ th dynamic system. The quantities  $\underline{\varepsilon}^\alpha(\underline{x})$  and  $\underline{l}^\alpha(\underline{x})$  are the stored energy density vector and the intensity vector, respectively, at the position vector  $\underline{x}$ . These vectorial quantities are associated with power flow toward junction  $\alpha$ .

Although superposition does not strictly hold for energetic (quadratic) quantities, nonetheless, under certain conditions and averagings, the superposition of these quantities may substantially hold [7]. Denoting the imposition of such conditions and the application of such averagings by triangular brackets, one may state the quantity  $\langle \underline{\varepsilon}(\underline{x}) \rangle$ , that relates to the stored energy vector, in the form

$$\langle \underline{\varepsilon}(\underline{x}) \rangle = \sum_{r, q} \langle (\underline{C}^\alpha)^{-1} \pi_\alpha(\underline{x}) \rangle . \quad (13)$$

[cf. equation (11).] Recently the authors derived the equation of the statistical energy analysis (SEA) using the procedure indicated in equation (13) [8, 9]. Similarly, one may state the quantities  $\langle \underline{l}(\underline{x}) \rangle$  and  $\langle \underline{\pi}(\underline{x}) \rangle$  that relate to the intensity vector and the net power flow vector, respectively, in the forms

$$\langle \underline{l}(\underline{x}) \rangle = \sum_{r, q} \underline{S}(\underline{x}_\alpha - \underline{x}_\beta) \langle (\underline{A}^\alpha)^{-1} \pi^\alpha(\underline{x}) \rangle , \quad (14)$$

$$\langle \underline{\pi}(\underline{x}) \rangle = \sum_{r, q} \underline{S}(\underline{x}_\alpha - \underline{x}_\beta) \langle \pi^\alpha(\underline{x}) \rangle , \quad (15)$$

where

$$\underline{\underline{S}}(x_\alpha - x_\beta) = (\text{sign}(x_{\alpha j} - x_{\beta j}) \delta_{ji}) \quad . \quad (16)$$

[cf. equations (12) and (6), respectively.] It is observed from equations (13) through (15), that whereas  $\langle \underline{\underline{E}}(X) \rangle$  involves the summation of the modified power flow quantities; one associated with propagations toward junction  $\alpha$  and the other toward junction  $\beta$ ,  $\langle \underline{\underline{L}}(X) \rangle$  and  $\langle \underline{\underline{T}}(X) \rangle$  involve the subtraction of the modified power flow quantities. The modifications of reference, when relevant, are stated in equations (11) and (12). Just as information, concerning a function, is complete only when its symmetric and anti-symmetric forms are on hand, so must one consider the information of the energetics of a complex grossly complete only when  $\langle \underline{\underline{E}}(X) \rangle$  and either or both  $\langle \underline{\underline{L}}(X) \rangle$  and  $\langle \underline{\underline{T}}(X) \rangle$ , are on hand. Barring the obvious exception, when either  $\pi^\alpha(X) \rightarrow 0$  or  $\pi^\beta(X) \rightarrow 0$ , the supplementarity of  $\langle \underline{\underline{E}}(X) \rangle$  and either or both  $\langle \underline{\underline{L}}(X) \rangle$  and  $\langle \underline{\underline{T}}(X) \rangle$  is thus suggested. Would one dare enter an acoustically driven room, without ear protection, merely on the basis that an intensity meter, on the average, reads zero?

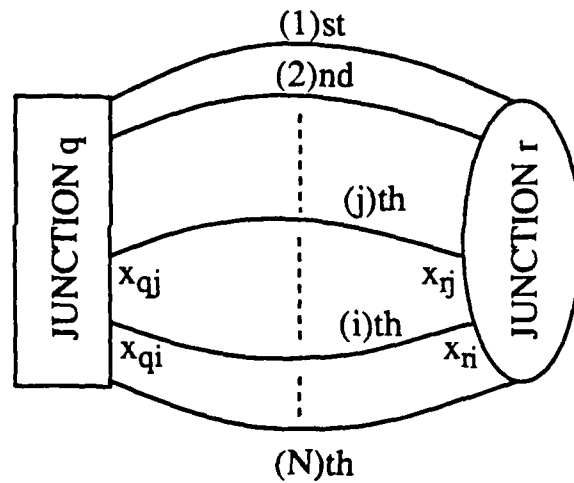


Fig. 1. A complex consisting of several one-dimensional dynamic systems. The dynamic systems terminate at two junctions, r and q. The couplings among the dynamic systems take place in the junctions.

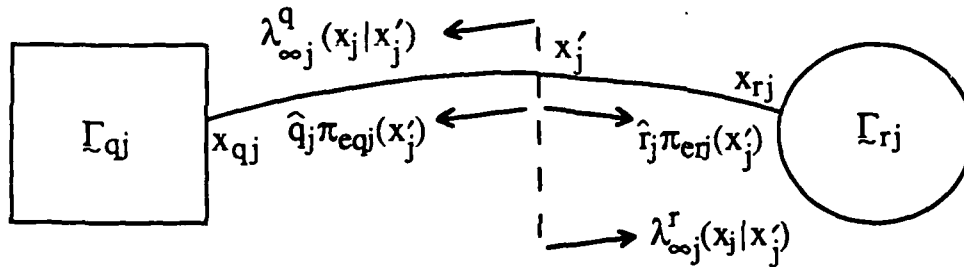


Fig. 2. Model of a typical elementary one-dimensional system. The essential properties of the dynamic system and the essential nature of the external input power are indicated.

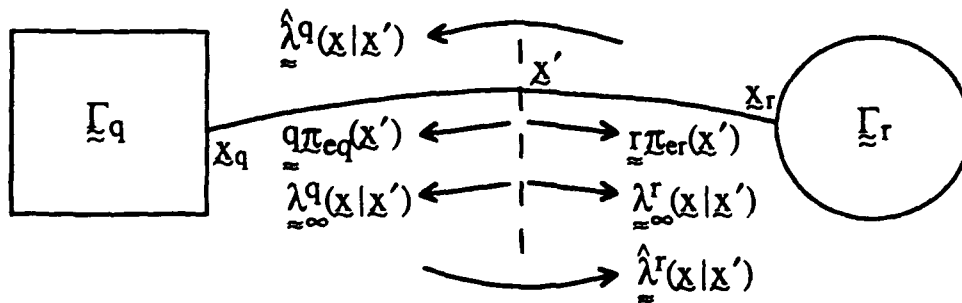


Fig. 3. A collective model of a complex composed of coupled one-dimensional dynamic systems. The terms in the power impulse response matrix, the terms in the external input power vector, the junction matrices, the terminal position vectors, and the external drive position vector, are indicated.

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